# Effects of the Skyrme tensor force on $0^{+}, 2^{+}$, and $3^{-}$states in ${ }^{16} \mathrm{O}$ and ${ }^{40} \mathrm{Ca}$ nuclei with second random phase approximation ${ }^{\dagger}$ 

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The effects of Skyrme tensor force on the natural parity $0^{+}, 2^{+}$, and $3^{-}$states of ${ }^{16} \mathrm{O}$ and ${ }^{40} \mathrm{Ca}$ were studied via the subtract second random-phase approximation (SSRPA) by adopting Skyrme energy density functionals (EDFs). To avoid the double counting and divergence in SRPA, the subtracting procedure was adopted in SSRPA calculations; consequently, SSRPA calculations can quickly converge with respect to the the energy cut-off. In the calculations, two tensor parameterizations, SGII+Te1 and SGII+Te3, were adopted, whose triplet-odd tensor terms have different signs, i.e., $U=-350.0 \mathrm{MeVfm}^{5}$ and $+200.0 \mathrm{MeVfm}^{5}$, respectively.
In the SRPA model, ${ }^{1,2)}$ the particle-hole ( $\mathrm{p}-\mathrm{h}$ ) excitation operator, $Q_{\nu}^{\dagger}$, can be written as,

$$
\begin{align*}
Q_{\nu}^{\dagger}= & \sum_{p h}\left(X_{p h}^{\nu} a_{p}^{\dagger} a_{h}-Y_{p h}^{\nu} a_{h}^{\dagger} a_{p}\right) \\
& +\sum_{\substack{p_{1}<p_{2} \\
h_{1}<h_{2}}}\left(X_{p_{1} p_{2} h_{1} h_{2}}^{\nu} a_{p_{1}}^{\dagger} a_{p_{2}}^{\dagger} a_{h_{2}} a_{h_{1}}\right.  \tag{1}\\
& \left.-Y_{p_{1} p_{2} h_{1} h_{2}}^{\nu} a_{h_{1}}^{\dagger} a_{h_{2}}^{\dagger} a_{p_{2}} a_{p_{1}}\right) .
\end{align*}
$$

Suffixes $p, p_{1}, p_{2}$ denote the particle states, while $h, h_{1}$, $h_{2}$ denote the hole states; $X$ and $Y$ denote forward and backward amplitudes, respectively. The SRPA equation has the same matrix form as the RPA equation:

$$
\left[\begin{array}{cc}
A & B  \tag{2}\\
-B^{*} & -A^{*}
\end{array}\right]\left[\begin{array}{c}
X^{\nu} \\
Y^{\nu}
\end{array}\right]=\hbar \omega_{\nu}\left[\begin{array}{c}
X^{\nu} \\
Y^{\nu}
\end{array}\right]
$$

where

$$
\begin{align*}
A & =\left(\begin{array}{ll}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{array}\right), B=\left(\begin{array}{ll}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{array}\right), \\
X & =\binom{X_{1}^{\nu}}{X_{2}^{\nu}}, Y=\binom{Y_{1}^{\nu}}{Y_{2}^{\nu}} . \tag{3}
\end{align*}
$$

Indices 1 and 2 denote the shorthand notations for 1 p 1 h and $2 \mathrm{p}-2 \mathrm{~h}$ configurations, respectively. Matrices $A_{12}, B_{12}$ represent the coupling of $1 \mathrm{p}-1 \mathrm{~h}$ and $2 \mathrm{p}-2 \mathrm{~h}$ configurations, and $A_{22}, B_{22}$ denote the coupling between 2p-2h configurations. In SSRPA, $\mathrm{A}_{11}$ and $\mathrm{B}_{11}$ are modified as:

$$
\begin{aligned}
& A_{11^{\prime}}^{S}=A_{11^{\prime}}+\sum_{2} A_{12}\left(A_{22}\right)^{-1} A_{21^{\prime}}+\sum_{2} B_{12}\left(A_{22}\right)^{-1} B_{21^{\prime}} \\
& B_{11^{\prime}}^{S}=B_{11^{\prime}}+\sum_{2} A_{12}\left(A_{22}\right)^{-1} B_{21^{\prime}}+\sum_{2} B_{12}\left(A_{22}\right)^{-1} A_{21^{\prime}}
\end{aligned}
$$

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Fig. 1. Isoscalar octupole strength distributions in ${ }^{16} \mathrm{O}$ calculated by RPA (upper panel) and SSRPA (lower panel). Black dashed, red solid, and blue dasheddotted lines represent the results of SGII, SGII+Te1, and SGII+Te3, respectively. Experimental data are denoted by $\left(E_{x}, B(E 3)\right)$ in units of MeV and $e^{2} \mathrm{fm}^{6}$.
where the diagonal approximation is adopted in $A_{22}$ matrix,

$$
\begin{equation*}
A_{22}^{D}=\delta_{p_{1} p_{1}^{\prime}} \delta_{p_{2} p_{2}^{\prime}} \delta_{h_{1} h_{1}^{\prime}} \delta_{h_{2} h_{2}^{\prime}}\left(E_{p_{1}}+E_{p_{2}}-E_{h_{1}}-E_{h_{2}}\right), \tag{4}
\end{equation*}
$$

while we fully take into account the coupling between the $2 \mathrm{p}-2 \mathrm{~h}$ configurations in $A_{22}$ in SRPA Eq. (2).

SSRPA calculations demonstrate a clear difference between the effects of two tensor interactions in SGII +Te 1 and SGII+Te3. The effects of triplet-even and triplet-odd tensor forces of SGII+Te3 on the $0^{+}$, $2^{+}$, and $3^{+}$states cancel each other and no significant effect is observed. In contrast, in the case of SGII +Te 1 , two tensor terms are added coherently, which improve remarkably the numerical results. Figure 1 presents an example of IS $3^{-}$excitations in ${ }^{16} \mathrm{O}$. These results indicate the importance of the triplet-odd and triplet-even terms for realistic calculations of collective states.
We adopted two extreme cases in this study to demonstrate how the energies and the strengths are affected by the tensor forces. Accordingly, we showed that the tense effects are observed even in the natural parity states of SSRPA results, while they are negligible in RPA, except the case of $3^{-}$. Thus, by setting appropriate tensor parameters, we will be able to improve further descriptions of the transition probabilities of low-lying collective states. This study remains for the future work.

## References

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2) C. Yannouleas, Phys. Rev. C 35, 1159 (1987).

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