$R = 12H_0^2$ and its implications to gravity and cosmology

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In the preceding article,[1] I report a model (QST model) that can yield the formulas of the standard model (SM) parameters. The model implies that the product of the Hubble constant H_0 and the Planck time t_{pl} is $H_0 t_{pl} = 2 \times (6\pi)^{-48} = \epsilon_0$. This relation is derived as follows.

In general relativity, the action is written as

$$S = \int \mathcal{L}\sqrt{-g}dx^4 = \int \mathcal{L}\sqrt{\det(g_{\mu\nu})}d^4x,$$

where $g_{\mu\nu}$ is the metric and $\mathcal{L}$ is the Lagrangian density of the system.

In the QST model, this is expressed as

$$(1+\hat{\mathcal{L}})\sqrt{-g} = 1 + \hat{\mathcal{L}} + \hat{\mathcal{L}}_{g3D}$$

where $\hat{\mathcal{L}}$ is the elementary action (EA), and $\hat{\mathcal{L}}_{g3D} = 3\epsilon_0$ is the EA corresponding to the gravity.

$$\sqrt{-g} = 1 + \hat{\mathcal{L}}_{g3D} = 1 + 3\epsilon_0$$

Now the spacetime metric of the Hubble expansion is

$$ds^2 = -dt^2 + e^{2H_0 t}(dx^2 + dy^2 + dz^2).$$

For $t = t_{\rm pl}$, we have $\sqrt{-g} = e^{3H_0 t} = 1 + 3H_0 t_{\rm pl}$. From the comparison, we have $H_0 t_{pl} = \epsilon_0$. The Ricci scalar curvature R and the 3D curvature K of this metric are

$$R = 12H_0^2,$$
$$K = 0.$$

Since $H_0 = \epsilon_0/t_{pl}$ is a constant, R is also a constant in the QST model.

Now, we generalize the metric to allow local changes of the scale with a constraint that Ricci curvature should remain constant.

$$ds^2 = -e^{2u(x,y,z)}dt^2 + e^{2H_0 t}(dx^2 + dy^2 + dz^2),$$
$$R = 12H_0^2.$$

The Ricci curvature of this metric is

$$R = -2(\Delta u + (\nabla u)^2)e^{-2H_0 t} + 12H_0^2 e^{-2u} = 12H_0^2.$$

We can show that Coulomb gravity can be derived from this metric. When we take the approximation $H_0 \simeq 0$, we have

$$R = -2(\Delta u + (\nabla u)^2) = 0.$$

Now, we define $\phi = e^u$; thus, we have

$$\Delta\phi = \Delta(e^u) = (\Delta u + (\nabla u)^2)\phi = 0,$$

which means that ϕ satisties the Laplace equation. The general solution of ϕ is

$$\phi(\vec{r}) = 1 - \Sigma_i \frac{r_s^i}{|\vec{r} - \vec{r_i}|}.$$

We note that $\phi^2 = e^{2u} = -g_{00}$. In general relativity, $g_{00} \simeq -1 - 2\phi_G$, where ϕ_G is the gravitational potential. This means that $\phi = \phi_G$. Thus, the relation $R = 0$ implies the Coulombic gravitational potential.

If we take into account the fact that $H_0 \neq 0$, the potential equation for gravity becomes non-linear.

$$\Delta\phi + 6H_0^2 e^{2H_0 t}(\phi - 1/\phi) = 0.$$

One may question the validity of the QST model, since the model implies that the Hubble constant is a constant, which would mean that spacetime expands exponentially at a constant rate. This contradicts standard Big Bang cosmology. In the Big Bang theory, H_0 is not constant, and the scale of the universe is zero at the Big Bang, which supposed to have occurred 13.8 billion years ago.

We note that there is no direct evidence that the Big Bang occured 13.8 billion years ago, because the absolute scale of the time coordinate in the past is impossible to determine. The distance to an object far away can only be measured by the red shift z. The value of z is then translated to the time based on standard cosmology by using the Friedmann equations. However, there is no experimental method to verify that the translation from z to t is correct.

The cosmic microwave background (CMB) is usually considered as the direct evidence of the Big Bang. I argue that it is not. CMB is definitive evidence that there was an era of $z = 1100$ when the spacetime scale is 1/1100 of the present one and the temperature of the Universe was 3000 K. In the standard cosmology, $z = 1100$ translates to 13.8 billion years ago and 380,000 years after the Big Bang. However, the only basis for this time scale is the Friedmann equations. If we assume that H_0 is constant, the only change is that the time of CMB formation becomes about 1 trillion years ago instead of 13.8 billion years ago.

The agreement between the abundance of ^{4}He and the Big Bang nucleosynthesis (BBN) model is considered another evidence of the Big Bang. In standard cosmology, BBN occured 3 minutes after the Big Bang. However, it is impossible to determine the absolute time when BBN occured. There is no observation that contradicts with the assumpition that H_0 is constant.

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References

1) Y. Akiba, in this report.