A model of the empirical mass formulae of elementary particles

Y. Akiba[*1]

In the preceding article[1)] empirical formulae of the masses of charged leptons (e, μ, τ), quarks (t, c, u, b, s, d), and gauge and Higgs bosons (Z, W, H) are presented. Here I report a model that can produce these mass formulae.

In the model, there is the minimum duration of time, which is the Planck time $\tau_{pl} = 1/M_{pl} = 5.3912 \times 10^{-44}$ sec. A term of the action (integral of Lagrangian) of a particle is a wedge product of 48 "primordial actions" (PAs) that are selected from the following 64 PAs:

$$S_{PA} = \Big\{ \frac{1}{6\pi} I^\mu \sigma^\nu \tau^a, \frac{1}{6\pi}\varepsilon i, \frac{1}{6\pi}\varepsilon\tau^3, \frac{1}{6\pi}\varepsilon I^c \tau^3, \\ -\frac{1}{2\pi} I^c, -\frac{1}{2\pi}\varepsilon i, i, I^c, -\varepsilon, \varepsilon, 1 \Big\}.$$

The wedge product operator $\wedge$ and orientation operator (sign operator) ε have the following reduction rules:

$$\hat\alpha \wedge \hat\beta = \begin{cases} \hat\alpha\hat\beta & (\hat\alpha\hat\beta = -\hat\beta\hat\alpha) \\ 0 & (\hat\alpha\hat\beta = \hat\beta\hat\alpha), \end{cases}$$

$$\hat\alpha\wedge\hat\beta\wedge\hat\gamma = \begin{cases} \hat\alpha\hat\beta\hat\gamma & (\hat\alpha\hat\beta = -\hat\beta\hat\alpha, \hat\beta\hat\gamma = -\hat\gamma\hat\beta, \hat\gamma\hat\alpha = -\hat\alpha\hat\gamma) \\ 0 & (otherwise), \end{cases}$$

$$s \wedge \hat\alpha = s\hat\alpha, s_1 \wedge s_2 = 0, \hat\alpha \wedge s \wedge \hat\beta = s\hat\alpha\hat\beta,$$
$$(\varepsilon\wedge)^n = 2^n.$$

Here, s is scalar. Following these rules, a 48 wedge product of PAs, $dS = \hat{s}_{i_1} \wedge \cdots \wedge \hat{s}_{i_{48}}$, is reduced to a reduced value in the form of $\pm m(6\pi)^n i^s (\tau^3)^{s'} \epsilon_0$, where $\epsilon_0 = 2 \times (6\pi)^{-48}$, $s \in \{0, 1\}$, $s' \in \{0, 1\}$, and m, $n (0 \le n \le 4)$ are integers. The values of s, s', n, and m are determined by the selection of the subset $\{\hat{s}_1, \cdots, \hat{s}_{48}\}$ from S_{PA}. The reduced values of 48 wedge products of PAs that correspond to the actions of charged leptons (e, μ, τ), neutrinos (ν_1, ν_2, ν_3), quarks (u, c, t, d, s, b), and bosons (Z, W, H) are summarized in Table 1. In the table, U and D stand for U-type and D-type quarks, respectively, when their mass is ignored.

The mass of particles can be obtained from this table. In the below, I shows how the mass formula of the electron can be obtained from the table as an example.

The action of the electron is

$$dS_e = 4(6\pi)^2 i\epsilon_0\tau^3 + i\epsilon_0 i\tau^3 + 3\epsilon_0.$$

Each term of the action can then be written as a product of the following "operators":

$$i\hat{d}_{16} = \epsilon_0^{1/3}(1 + i\sigma^1 + i\sigma^2 + i\sigma^3),$$
$$\hat\psi_{15} = |\psi_{15}|\tau^2(u_1^2\sigma^1 + u_2^2\sigma^2 + u_3^2\sigma^3),$$
$$\hat\psi_{15}^\dagger = |\psi_{15}|\tau^1(u_1^1\sigma^1 + u_2^1\sigma^2 + u_3^1\sigma^3),$$
$$\hat{A}_{18} = (6\pi)^{-2}\epsilon_0^{1/3}(1 + i\sigma^1 + i\sigma^2 + i\sigma^3).$$

Here, $|\psi_{15}| = (6\pi)\epsilon_0^{1/3}$ and the vectors $u^1 = (u_1^1, u_2^1, u_3^1)$ and $u^2 = (u_1^2, u_2^2, u_3^2)$ satisfy $u^1 \times u^2 = (-1, -1, -1)$ and $u^1 \cdot u^2 = 1$. The operators id_{16} and ψ_{15} correspond to the differential operator $i\sigma^\mu\partial_\mu$ and the electron field operator, and $\psi_{15}^\dagger$ corresponds to its conjugate. One can show that

$$i\hat{d}_{16}\hat\psi_{15}^\dagger\hat\psi_{15} = 4(6\pi)^2\epsilon_0 i\tau^3,$$
$$\hat{A}_{18} \wedge \hat\psi_{15}^\dagger\hat\psi_{15} = \epsilon_0 i\tau^3,$$
$$\mu_e^0|\psi_{15}|^2 = 3\epsilon_0.$$

Here, $\mu_e^0 = (12\pi^2)^{-1}\epsilon_0^{1/3}$. The terms $4(6\pi)^2\epsilon_0 i\tau^3$, $\epsilon_0 i\tau^3$, and $3\epsilon_0$ correspond to the kinetic, the EM interaction, and the mass term, respectively. From these relations, the mass formula of the electron can be determined.

$$dS_e = \left(1 + \frac{1}{4}\frac{1}{(6\pi)^2}\right)\left(i\hat{d}_{16}\hat\psi_{15}^\dagger\hat\psi_{15} + \mu_e|\psi_{15}|^2\right),$$
$$\mu_e = \left(1 + \frac{1}{4}\frac{1}{(6\pi)^2}\right)^{-1}\frac{1}{12\pi^2}\epsilon_0^{1/3}.$$

Similarly, the mass formulae of μ, τ, ν_1, ν_2, ν_3, u, d, s, c, b, t, Z, W, and H can be obtained from each row in the Table 1.

Table 1. The action of particles per Planck Time.

	$(6\pi)^2\epsilon_0$	$(6\pi)\epsilon_0$				ϵ_0		
	$i\tau^3$	$i\tau^3$	i	τ^3	1	$i\tau^3$	i	1
e	4					1		3
μ	4	−12				27		3
τ	4	−3		3		1+4		
ν_1	−12	−12						−2
ν_2	4		−4					2
ν_3	−12	−12			±2			
U	−12					±2		
u	−4							±8
c	−4							±24
t	−4							±8
D	−12					1		
d	−12	−12		3				
s	−12							3
b	4	6		3		27	18	
Z	12	1				1	18	
H	4	6					±18	
W	12		18			27	18	

*1 RIKEN Nishina Center

Reference

1) Y. Akiba, RIKEN Accel. Prog. Rep. **52** (2018).