The $\pi\gamma \to \pi\pi$ transition and the ρ radiative decay width from lattice QCD†

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Lattice QCD calculations of hadronic matrix elements of external currents are straightforward as long as the initial and final states contain no more than a single, stable hadron. For multi-hadron states, however, the relation between the finite-volume matrix elements computed on the lattice and the physical infinite-volume matrix elements of interest is quite nontrivial, and is known only for certain cases. The formalism for $1 \to 2$ transition matrix elements was pioneered by Lellouch and Lüscher in 2000,[4)] and was later generalized by other authors to more complicated systems.[5–10)]

Our collaboration is using this formalism to compute several $1 \to 2$ transition matrix elements of interest in high-energy and nuclear physics, including semileptonic weak decays such as $B \to \pi\pi\ell\bar{\nu}$. The present work considers the electromagnetic process $\pi\gamma \to \pi\pi$, where we take the $\pi\pi$ system in a P wave and isospin 1, and allow the photon to be virtual. The hadronic matrix element for this process can be written as

$$\langle\pi\pi|J^\mu|\pi\rangle = \frac{2i\mathcal{V}(q^2,\, s)}{m_\pi}\epsilon^{\nu\mu\alpha\beta}\epsilon_\nu(P,\, m)(p_\pi)_\alpha P_\beta, \quad (1)$$

where P and ϵ are the four-momentum and polarization of the two-pion final state, p_π is the four-momentum of the single-pion initial state, and the amplitude $\mathcal{V}(q^2,\, s)$ depends on the two scalar variables $q^2 = (p_\pi - P)^2$ and $s = P^2$. Our calculation was performed with $2 + 1$ flavors of clover fermions, at a pion mass of approximately 320 MeV. Our results for $\mathcal{V}(q^2,\, s)$ are shown in Fig. 1. This amplitude shows the expected enhancement associated with the ρ resonance, which corresponds to a pole at $s_{\rm pole} \approx m_\rho^2 + im_\rho\Gamma_\rho$. One very interesting result, seen for the first time, is the following: for large s, $\mathcal{V}(q^2,\, s)$ falls off significantly slower compared to what one would expect for purely resonant behavior.

The residue of $\mathcal{V}(0,\, s)$ at $s = s_{\rm pole}$ is equal to the product of the ρ-$\pi\pi$ and ρ-$\pi\gamma$ couplings. Our result for the photocoupling is $|G_{\rho\pi\gamma}| = 0.0802(32)(20)$, where the first uncertainty originates from the two-point and three-point function fits, while the second uncertainty is an estimate of the parametrization dependence in the analytic continuation. Despite the heavier-than-physical light-quark masses, the lattice result for $|G_{\rho\pi\gamma}|$ is already close to the value extracted from the measured ρ radiative decay width.[11)]

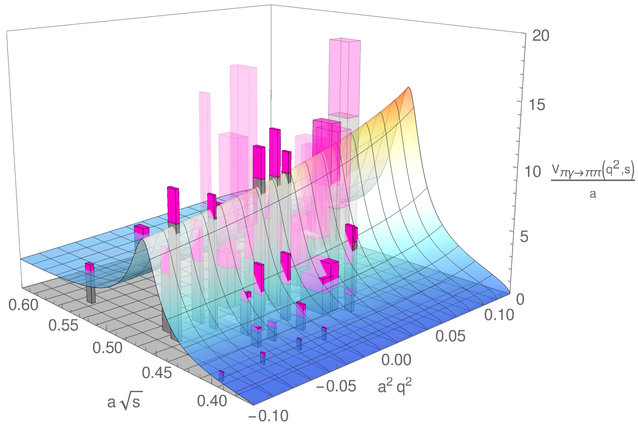

Fig. 1. Our results for the $\pi\gamma \to \pi\pi$ transition amplitude as a function of the $\pi\pi$ invariant mass and the photon virtuality. The magenta boxes show the 1σ uncertainties in $a\sqrt{s}$, a^2q^2, and $\mathcal{V}/a$.

† Condensed from the article in Phys. Rev. D **98**, 074502 (2018)

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References

1) M. Lüscher, Nucl. Phys. B **354**, 531 (1991).
2) R. A. Briceño, J. J. Dudek, R. D. Young, Rev. Mod. Phys. **90**, 025001 (2018).
3) C. Alexandrou *et al.*, Phys. Rev. D **96**, 034525 (2017).
4) L. Lellouch, M. Lüscher, Commun. Math. Phys. **219**, 31 (2001).
5) C. J. D. Lin, G. Martinelli, C. T. Sachrajda, M. Testa, Nucl. Phys. B **619**, 467 (2001).
6) N. H. Christ, C. Kim, T. Yamazaki, Phys. Rev. D **72**, 114506 (2005).
7) M. T. Hansen, S. R. Sharpe, Phys. Rev. D **86**, 016007 (2012).
8) R. A. Briceño, Z. Davoudi, Phys. Rev. D **88**, 094507 (2013).
9) R. A. Briceño, M. T. Hansen, A. Walker-Loud, Phys. Rev. D **91**, 034501 (2015).
10) R. A. Briceño, M. T. Hansen, Phys. Rev. D **92**, 7, 074509 (2015).
11) C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C **40**, 100001 (2016).