The chiral propulsion effect†

Y. Hirono,*3 D. E. Kharzeev,*1,*2,*3 and A. V. Sadofyev*4

The physics of chiral media has attracted a significant attention recently. Remarkably, it appears that the quantum chiral anomaly significantly affects the macroscopic behavior of chiral media and induces new transport phenomena,[1,2] such as the Chiral Magnetic and Chiral Vortical Effects (CME and CVE, respectively). CME and CVE refer to the generation of electric currents along an external magnetic field or vorticity in the presence of a chirality imbalance. The resulting currents are non-dissipative due to the protection by the global topology of the gauge field. These chiral effects are expected to occur in a variety of systems: the quark-gluon plasma, Dirac and Weyl semimetals, primordial electroweak plasma, and cold atoms. In quark-gluon plasma, the chirality imbalance can be produced by topological fluctuations of QCD, or by the combination of electric and magnetic fields that accompany heavy-ion collisions.

Consider the motion of a vortex filament in a fluid. It can be described by the localized induction equation (LIE),

$$\dot{\boldsymbol{X}} = C\boldsymbol{X}' \times \boldsymbol{X}'', \tag{1}$$

where $\boldsymbol{X} = \boldsymbol{X}(t, s)$ denotes the position of a vortex, t is the time, s is the arc-length parameter, the dot and the prime indicate the derivatives with respect to t and s respectively, and C is a parameter dependent on the properties of the fluid. Interestingly, the LIE (1) can be mapped to the non-linear Schrödinger equation (NLSE) by the so-called Hasimoto transformation,

$$\psi(t, s) = \kappa(t, s) \exp\left[i \int^s \tau(t, s')ds'\right], \tag{2}$$

where $\kappa(t, s)$ is the curvature and $\tau(t, s)$ is the torsion of a vortex. NLSE is known to be a completely integrable system which has solitonic solutions and an infinite sequence of commuting conserved charges. The LIE possesses solutions that represent helical excitations propagating along the vortex; they are known as Hasimoto solitons.

Let us now consider a system in which parity is broken by the presence of magnetic helicity; the corresponding term in the action is

$$S_\chi = \int dt\, \mu\, \mathcal{H}, \tag{3}$$

where μ is the "chiral" chemical potential, $\mathcal{H}$ is the magnetic helicity given by $\mathcal{H} = \frac{e^2}{4\pi^2}\int d^3x\, \boldsymbol{A}\cdot\boldsymbol{B}$, where $\boldsymbol{A}$ is the vector potential and $\boldsymbol{B}$ is the magnetic field. It is worth mentioning that taking the derivative of this action with respect to the vector potential, one readily finds the CME current: $\boldsymbol{J}_{\mathrm{CME}} = \delta S_\chi/\delta\boldsymbol{A} \propto \boldsymbol{B}$. Supplementing the non-relativistic Abelian Higgs model with the term given by Eq. (3), one can find the equation of motion for a quantized magnetic vortex at finite μ, as derived by Kozhevnikov:

$$\dot{\boldsymbol{X}} = C\boldsymbol{X}' \times \boldsymbol{X}'' + \mu\left[\boldsymbol{X}''' + \frac{3}{2}(\boldsymbol{X}'')^2\boldsymbol{X}'\right], \tag{4}$$

where a tangential term $\frac{3}{2}\mu(\boldsymbol{X}'')^2\boldsymbol{X}'$ is added to keep the arc-length-preserving property.

In this paper we are interested in the behavior of chiral solutions. We can find a simple explicit solution of the FME (4) having the form of a helix,

$$\boldsymbol{X}_{\mathrm{helix}}(t, s) = \frac{1}{A^2}\begin{pmatrix}\kappa_0\cos[A(s - v_p t)]\\ \kappa_0\sin[A(s - v_p t)]\\ \tau_0\, A(s - v_g t)\end{pmatrix}, \tag{5}$$

where the constants κ_0 and τ_0 give the curvature and the torsion of the helix, $A = \sqrt{\kappa_0^2 + \tau_0^2}$, and the phase and group velocities are given by $v_p = \tau_0 + \mu(\tau_0^2 - \frac{\kappa_0^2}{2})$, $v_g = -\frac{\kappa_0^2}{\tau_0} - \frac{3\kappa_0^2}{2}\mu$. Note that the sign of τ_0 determines the handedness of the helix. The radius R and the pitch ℓ of the helix are given by $R = \kappa_0/(\kappa_0^2 + \tau_0^2)$, $\ell = 2\pi\tau_0/(\kappa_0^2 + \tau_0^2)$.

Using the map between the FME and the Hirota equation, we find a propagating solitonic solution of the FME,

$$\boldsymbol{X}_{\mathrm{sol}}(t, s) = \begin{pmatrix}-\frac{2\epsilon}{\epsilon^2+\tau_0^2}\ \mathrm{sech}[\epsilon\xi]\cos[\eta]\\ -\frac{2\epsilon}{\epsilon^2+\tau_0^2}\ \mathrm{sech}[\epsilon\xi]\sin[\eta]\\ s - \frac{2\epsilon}{\epsilon^2+\tau_0^2}\tanh[\epsilon\xi]\end{pmatrix}. \tag{6}$$

where $\eta \equiv \tau_0\, s + (\epsilon^2 - \tau_0^2)t + \mu\tau_0(3\epsilon^2 - \tau_0^2)$, $\xi \equiv s - (2\tau_0 + \mu\left(3\tau_0^2 - \epsilon^2\right))t$, and ϵ and τ_0 are constants. This soliton has a constant torsion given by τ_0 and propagates in the z direction. Its speed is modified by μ and reduces to the original Hasimoto soliton at $\mu = 0$.

To summarize, we have found that in chirally imbalanced media there exist helical excitations that carry energy along the vortex (the Chiral Propulsion Effect).

References

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*1 RIKEN Nishina Center
*2 Department of Physics and Astronomy, Stony Brook University
*3 Department of Physics, Brookhaven National Laboratory
*4 Los Alamos National Laboratory