Origin of the fake eigen energy of the two-baryon system in lattice QCD†

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Both the direct and HAL QCD methods are used to study two-hadron systems in lattice QCD. In previous studies for large pion masses,[2)] the direct method showed that both dineutron and deuteron are bound. However, the HAL QCD method suggests that these are unbound. In the series of papers,[3,4)] we pointed out that these discrepancies originate from the misidentification of the ground state in the direct method due to the scattering states,[3)] which can be revealed by some simple tests using Lüscher's finite volume formula.[4)]

In the direct method, one measures the energy eigenvalue. It is estimated by the plateau value of the effective energy shift, which is given by

$$\Delta E_{\mathrm{eff}}(t) \equiv \frac{1}{a} \log \left[\sum_{\vec{r}} R(\vec{r},t)\right] \Big/ \left[\sum_{\vec{r}} R(\vec{r},t+a)\right] \quad (1)$$

using the R-correlator

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{C_B(t)\}^2}, \quad (2)$$

where $\mathcal{J}(B)$ is a source(sink) operator and the baryon propagator $C_{\mathrm{B}}(t) \equiv \langle B(t)\bar{B}(0)\rangle$. It converges to the ground state energy at a large time, where the ground state is saturated. For example, the inelastic state becomes negligible around 1 fm, while the elastic excitation in the two-baryon system remains even around $\mathcal{O}(10)$ fm, which causes a fake plateau-like structure around 1.5 fm in the actual calculations.

Such a fake plateau problem can be checked by the source dependence.[3)] Figure 1 shows the effective energy shift of $\Xi\Xi(^1\mathrm{S}_0)$ at $m_\pi = 0.51$ GeV using the wall and the smeared sources. There is a plateau-like structure around $t \sim 15a \simeq 1.5$ fm, but it depends on the source, which means either (or both) of the results is fake.

Since the time-dependent HAL QCD method uses both the ground and the scattering states simultaneously to extract the interaction, it does not require the ground-state saturation. In this method, the potential is defined from the R-correlator, and some systematic uncertainties are shown to be under control.[1)]

Using the correct eigen energies ΔE_n and eigenfunction $\Psi_n(r)$, which are obtained by solving $H \equiv H_0 + V(r)$ with the HAL QCD potential $V(r)$ in the finite box, the R-correlator is expanded by

$$R(\vec{p}=0,t) \simeq \sum_{\vec{r}}\sum_{n} a_n \Psi_n(\vec{r}) e^{-\Delta E_n t} = \sum_{n} b_n e^{-\Delta E_n t} \quad (3)$$

† Condensed from the article in Proceedings for the 35th International Symposium on Lattice Field Theory[1)]

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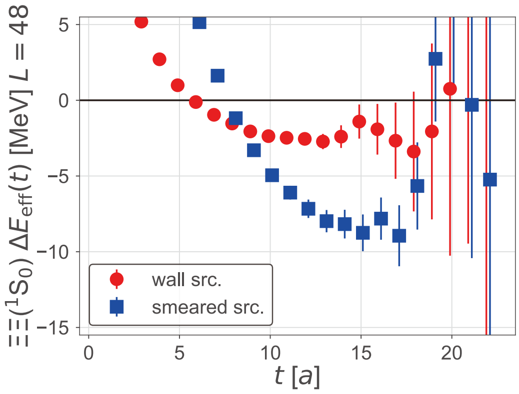

Fig. 1. The effective energy shift using the wall and the smeared source for $\Xi\Xi(^1\mathrm{S}_0)$ at $m_\pi = 0.51$ GeV. The lattice size $L = 48$ with the lattice spacing $a \simeq 0.09$ fm.

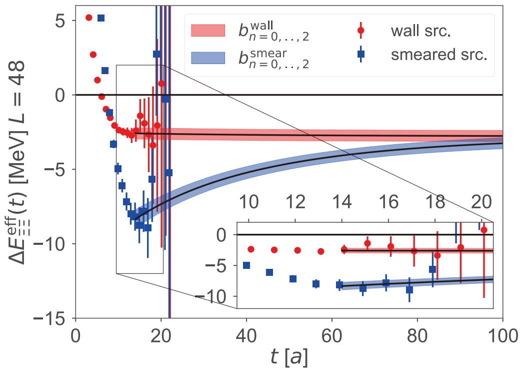

Fig. 2. Reconstructed $\Delta E_{\mathrm{eff}}(t)$ and its convergence.

The contamination coefficients b_n are determined from the orthogonality of $\Psi_n(\vec{r})$.

Figure 2 shows the $\Delta E_{\mathrm{eff}}(t)$ reconstructed using a low-lying b_n and ΔE_n, which well reproduces the fake plateau. The ground-state saturation of the smeared source is estimated to be around $t \sim 100a \sim 10$ fm at $L = 48$. This result proves the advantages of the HAL QCD method, and the direct measurement of the two-baryon system is not practical.

References

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