Stability of the wobbling motion in an odd-A nucleus[†]

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Recently, the transverse wobbling mode was proposed in the yrast band near the ground state before the first backbending in ^{135}Pr[1)]. The transverse wobbling mode is the wobbling motion around the middle moment of inertia (MoI)[2)], which does not exist in the pure rotor as discussed in the context of classical mechanics[3)] and by Bohr-Mottelson[4)] quantum mechanically. Regarding the particle-rotor model, as long as the rigid MoI is adopted, there is no chance to find transverse wobbling, because the single-particle oscillator strength ω_k and the rigid MoI are derived from a common radius, and their magnitudes increase or decrease in the same direction as functions of γ periodically with a span of $2\pi/3$. On the other hand, the hydrodynamical (hyd) MoI changes its role in every span of $\pi/3$ in an opposite direction to ω_k. Thus, there remains a possibility to find the transverse wobbling mode for the particle-rotor model with hyd MoI.

We extend the Holstein-Primakoff (HP) boson expansion method to the odd-A case[5–8)] by introducing two kinds of bosons for the total angular momentum $\vec{I}$ and the single-particle angular momentum $\vec{j}$. We can identify the nature of each band by referring to two kinds of quantum numbers (n_α, n_β) which indicate the wobbling of $\vec{I}$ and the precession of $\vec{j}$, respectively. In this paper we extend this method to the particle-rotor model with hyd MoI. We choose a representation in which I_y and j_x are diagonal because the hyd MoI is maximum around the y-axis with the relation $\mathcal{J}_y^{\rm hyd} \geq \mathcal{J}_x^{\rm hyd} \geq \mathcal{J}_z^{\rm hyd}$ in the range of $0 \leq \gamma \leq \pi/6$, while ω_k favors $\vec{j}$ to align along the x-axis in the same range of γ. We notice that, if we choose the diagonal representation for $\vec{I}$ and $\vec{j}$ in the same direction, we cannot find any stable physical solution for common values of γ and V (the strength of the single-particle potential[5–8)]) in the range of $11/2 \leq I \leq 33/2$. We solve the energy-eigenvalue equation to obtain two real solutions, i.e., ω_+, which is the higher energy, and ω_-, which is the smaller one. In the symmetric limit of $\gamma = 30°$ and $V = 0$, ω_+ corresponds to the wobbling motion around the y-axis with the maximum MoI, while ω_- corresponds to the precession of j around the x-axis.

We adopt j =11/2, $\mathcal{J}_0$ =25 MeV^{-1} ($\mathcal{J}_k^{\rm hyd} = \frac{4}{3}\mathcal{J}_0 \sin^2\left(\gamma + \frac{2}{3}\pi k\right)$), β =0.18 and γ =26° (proposed by Ref.[1)]), and V = 1.6 MeV (related to the single-particle strength with these β, γ and j by Wigner-Eckart theorem)[8)]. In Fig. 1 we compare the energies labeled by (n_α, n_β)= (0,1) and (0,0) with the exact ones obtained by diagonalizing the same Hamiltonian. The exact levels are reproduced by the approximate ones labeled (0,1), the precession mode of j, irrespective of $I-j$. Both ω_+ and ω_- monotonically increase with I, and never decrease.

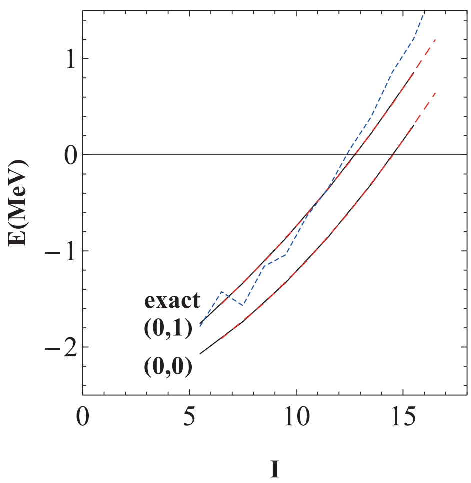

Fig. 1. Comparison of the excitation energy in the leading order approximation[5)] with the exact results as functions of I. The solid lines correspond to $I-j$ =even, the red dashed lines to $I-j$ =odd, and the dotted blue lines to the exact results. The attached numerals (0,1) and (0,0) correspond to quantum numbers of (n_α, n_β).

In conclusion, there is no transverse wobbling mode within the framework of the particle-rotor model even with the hyd MoI.

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