Gauge symmetry in the large-amplitude collective motion of superfluid nuclei†

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The adiabatic self-consistent collective coordinate (ASCC) method[2)] is a practical method for describing large-amplitude collective motion in atomic nuclei with superfluidity and an advanced version of the adiabatic time-dependent Hartree–Fock–Bogoliubov theory. In the application of the one-dimensional ASCC method, Hinohara et al.[3)] encountered numerical instability and found that it was caused by the symmetry of the basic equations of the ASCC method under a certain continuous transformation. This transformation involves the gauge angle φ and changes the phase of the state vector. In this sense, Hinohara et al. called it the "gauge" symmetry. They proposed a gauge-fixing prescription to remove redundancy associated with the gauge symmetry and successfully applied it to the shape coexistence phenomena in proton-rich Se and Kr isotopes.

We investigated this symmetry on the basis of the Dirac–Bergmann theory of constrained systems[4,5)]. As is well known, the gauge symmetry is associated with constraints originating from the singularity of the Lagrangian. In the ASCC method, the linear term of the particle number n in the collective Hamiltonian can be regarded as a constraint, and it leads to the gauge symmetry.

In the ASCC method, we assume the following form of the state vector.

$$|\phi(q,p,\varphi,n)\rangle = e^{-i\varphi\tilde{N}}|\phi(q,p,n)\rangle = e^{-i\varphi\tilde{N}}e^{i\hat{G}}|\phi(q)\rangle,$$

with $\hat{G}(q,p,n) = p\hat{Q}(q) + n\hat{\Theta}(q)$ and $\tilde{N} = \hat{N} - N_0$. φ is the gauge angle conjugate to the particle number $n = N - N_0$ measured from a reference value N_0. The collective Hamiltonian is defined and expanded up to $O(n)$ as below.

$$\mathcal{H}(q,p,n) := \langle\phi(q,p,\varphi,n)|\hat{H}|\phi(q,p,\varphi,n)\rangle = V(q) + \frac{1}{2}B(q)p^2 + \lambda n. \quad (1)$$

This can be regarded as a system with the constraint $n = 0$, and λ is a Lagrange multiplier.

In Lagrange formalism, the Lagrangian corresponding to this (total) Hamiltonian is given by

$$L = \frac{1}{2B(q^1)}(\dot{q}^1)^2 - V(q^1,q^2), \quad (2)$$

with $(q^1,q^2) := (q,\varphi)$, and the rank of the Hessian $(\partial^2 L/\partial\dot{q}^i\partial\dot{q}^j)$ is one. (We allowed the potential V to depend on $q^2 = \varphi$ in order to make it easy to observe the number of the degrees of freedom.) Hence, this Lagrangian leads to one constraint, $p_2 = \frac{\partial L}{\partial \dot{q}^2} = n = 0$. The time derivative of the constraint is given by $\dot{n} = \{n,\mathcal{H}\} = -\partial_\varphi V$. Thus, the constraint is preserved in time if $V(q,\varphi) = V(q)$. Then, we have only one constraint $n = 0$, and it is a first-class constraint. From the above, it is clear that our system has one gauge degree of freedom.

It is known that a generator of a gauge transformation can always be written as a "linear combination" of the first-class constraints. We can write the generator G as $G = \epsilon(q,p,\varphi,n,t)n$ with an infinitesimal function ϵ. This generator gives the gauge transformation of collective variables.

$$\delta q = n\partial_p\epsilon \approx 0, \quad \delta p = -n\partial_q\epsilon \approx 0, \quad (3)$$

$$\delta\varphi = \epsilon + n\partial_n\epsilon \approx \epsilon, \quad \delta n = -n\partial_\varphi\epsilon \approx 0. \quad (4)$$

Here, the symbol $\approx$ denotes weak equality. As ϵ is an arbitrary function of (q,p,φ,n,t), in particular, of p, the linear and higher-order terms of p are mixed only into φ by this gauge transformation. This is important for the adiabatic expansion in the ASCC method to make sense.

With the choice $\epsilon = \alpha pn$, we obtain

$$\delta q = \alpha n, \quad \delta p = 0, \quad \delta\varphi = \alpha p, \quad \delta n = 0, \quad (5)$$

which leads to the transformation of operators,

$$\hat{Q} \to \hat{Q} + \alpha\tilde{N}, \quad \hat{\Theta} \to \hat{\Theta} + \alpha\hat{P}. \quad (6)$$

This is exactly the transformation found in Ref. 3), and thus, we confirmed that the symmetry discussed in Ref. 3) is a gauge symmetry.

In Ref. 1), the most general gauge transformation is discussed. While the equation of collective submanifold, from which the basic equations of the ASCC method (the moving-frame HFB & QRPA equations) are derived, is invariant under the most general gauge transformation, the gauge symmetry is partially broken by the adiabatic expansion at the level of the moving-frame HFB & QRPA equations. Above, we have considered the expansion of the collective Hamiltonian up to $O(n)$. In Ref. 1), it is also shown that there is no gauge symmetry in the case where the collective Hamiltonian is expanded up to $O(n^2)$.

† Condensed from the article in Ref. 1)

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References

1) K. Sato, Prog. Theor. Exp. Phys. **2015**, 123D01
2) M. Matsuo, T. Nakatsukasa, and K. Matsuyanagi, Prog. Theor. Phys. **103**, 959 (2000).
3) N. Hinohara, T. Nakatsukasa, M. Matsuo, and K. Matsuyanagi, Prog. Theor. Phys. **117**, 27 (2007).
4) P. A. M. Dirac, Can. J. Math. **2**, 129 (1950).
5) J. L. Anderson, P. G. Bergmann, Phys. Rev. **83**, 1018 (1951).