Dipolar quantization and the infinite circumference limit of 2d CFT†

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We elaborate on our previous work[1)], where a new term *dipolar quantization* was introduced, and further argue that adopting $L_0 - (L_1 + L_{-1})/2$ as the Hamiltonian yields an infinite circumference limit in two-dimensional conformal field theory. One of the physical significances of this new Hamiltonian is that the time translational vector field coincides with the electric field of an electric dipole located at $z = 1$, as opposed to in the ordinary radial quantization case. The new theory then exhibits a continuous and strongly degenerated spectrum as well as the Virasoro algebra with a continuous index.

First, let us define the following "charges":

$$\mathcal{L}_\kappa \equiv \frac{1}{2\pi i} \oint^t dz g(z) f_\kappa(z) T(z), \tag{1}$$

where $T(z) = T_{zz}(z)$ is the holomorphic part of the energy-momentum tensor of the original CFT. Further, the integration is performed along the path where the Euclidean time t assumes a constant value. The relationship between the Euclidean time t and the z coordinate is best summarized by introducing a new coordinate w as

$$w = t + is, \tag{2}$$

where s represents the Euclidean space part. Then, the following relations are imposed:

$$\frac{\partial w}{\partial z} = \frac{1}{g(z)}, \tag{3}$$

$$f_\kappa(z) = e^{\kappa w}. \tag{4}$$

Now we can define $\mathcal{L}_\kappa$ by choosing an appropriate value of $g(z)$. In particular, for $\kappa = 0$, we have the expression for the Hamiltonian for $g(z)$ below corresponding to the case for the ordinary radial quantization and for the dipolar quantization:

$$\mathcal{L}_0 = \frac{1}{2\pi i} \oint^t dz g(z) T(z) = \begin{cases} L_0 & \text{for } g(z) = z \\ L_0 - \frac{L_1 + L_{-1}}{2} & \text{for } g(z) = -\frac{(z-1)^2}{2} \end{cases}. \tag{5}$$

One can then calculate the commutation relations between $\mathcal{L}_\kappa$'s as follows:

$$[\mathcal{L}_\kappa, \mathcal{L}_{\kappa'}] = (\kappa - \kappa')\mathcal{L}_{\kappa+\kappa'} + \begin{cases} \frac{c}{12}(\kappa^3 - \kappa)\delta_{\kappa+\kappa',0} & \text{for } g(z) = z \\ \frac{c}{12}\kappa^3 \delta(\kappa + \kappa') & \text{for } g(z) = -\frac{(z-1)^2}{2} \end{cases}. \tag{6}$$

Here, for $g(z) = -\frac{(z-1)^2}{2}$, κ assumes all the real values, while κ is an integer for $g(z) = z$. By using this commutation relation with the Hamiltonian

$$H = \mathcal{L}_0 + \bar{\mathcal{L}}_0, \tag{7}$$

where $\bar{\mathcal{L}}_0$ stands for a member of another set of similar "charges", one can construct a state with an arbitrary value of energy in the following manner. First, consider an eigenstate of $\mathcal{L}_0$ with an eigenvalue α and with an additional index σ labeling a possible degeneracy:

$$|\alpha, \sigma\rangle, \tag{8}$$

so that

$$\mathcal{L}_0|\alpha, \sigma\rangle = \alpha|\alpha, \sigma\rangle. \tag{9}$$

In this case, operating on $|\alpha, \sigma\rangle$ with $\mathcal{L}_\kappa$ yields

$$\mathcal{L}_\kappa|\alpha, \sigma\rangle = |\alpha - \kappa, \sigma\rangle. \tag{10}$$

Thus, starting from the vacuum or any other energy eigenstate, we can construct an eigenstate for $\mathcal{L}_0$ with an arbitrary eigenvalue because κ can assume any real value.

Further, one can show that

$$\mathcal{L}_\kappa^\dagger = \mathcal{L}_{-\kappa}, \tag{11}$$

where † stands for the Hermitian conjugation. One of the idiosyncrasies we find in this formulation is a different inner product of the Hilbert space. For the choice of $g(z) = -\frac{(z-1)^2}{2}$, the following holds:

$$(L_{-1})^\dagger = L_{-1}, \quad (L_0)^\dagger = 2L_{-1} - L_0, \quad (L_1)^\dagger = L_1 - 4L_0 + 4L_{-1}. \tag{12}$$

Equation (13) shows that the operations of the Hermitian conjugation operations in dipolar quantization on L_{-1}, L_0 and L_1 are closed among themselves. In addition, they definitely assume a different form from those in radial quantization. Nonetheless, if we compute the Hermitian conjugate for the combination $L_0 - (L_1 + L_{-1})/2$, which is the Hamiltonian for dipolar quantization, it proves to be Hermitian (in the sense of dipolar quantization):

$$\left(L_0 - \frac{L_1 + L_{-1}}{2}\right)^\dagger = 2L_{-1} - L_0 - \frac{L_1 - 4L_0 + 4L_{-1} + L_{-1}}{2} = L_0 - \frac{L_1 + L_{-1}}{2}. \tag{13}$$

Thus, we presented a new conformal symmetric quantum system with novel Hamiltonian and Hilbert space.

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References

1) N. Ishibashi and T. Tada, J. Phys. A **48**, no. 31, 315402 (2015) [arXiv:1504.00138 [hep-th]].