Lattice determination of $|V_{us}|$ with inclusive hadronic τ decay experiment[†]

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The Kobayashi-Maskawa matrix element $|V_{us}|$ is an important parameter for flavor physics, which is relevant for searching new physics beyond the standard model in particle physics. Thus far, $|V_{us}|$ has been most precisely determined by kaon decay experiments. As an alternative, $|V_{us}|$ can also be determined independently from τ decay. A conventional method is to use the so-called finite energy sum rule with polynomial weight function $\omega(s)$ and spectral function $\rho^{(J)}_{V/A}$ with spin $J = 0, 1$ as

$$\int_0^{s_0} \omega(s)\rho(s)ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} \omega(s)\Pi(s)ds, \qquad (1)$$

where $\Pi(s)$ is a hadronic vacuum polarization (HVP) function. Here, $\rho(s)$ on the left hand side is related to the differential τ decay rate by hadronic V and A currents with u, s flavors as

$$\frac{dR_{us;V/A}}{ds} = \frac{12\pi^2|V_{us}|^2 S_{EW}}{m_\tau^2}(1-y_\tau)^2 \times \left[(1+2y_\tau\rho^{(0+1)}_{us;V/A} - 2y_\tau\rho^0_{us;V/A})\right], \qquad (2)$$

where $y_\tau = s/m_\tau^2$, and S_{EW} is a known short-distance electroweak correction. The HVP function $\Pi(s)$ on the right hand side in Eq.(1) is analytically calculated using OPE based on perturbative QCD (pQCD). Thus, the value of momentum s_0 should be sufficiently large to use a perturbative OPE result. By combining both the inclusive τ decay experiments and pQCD, one can obtain $|V_{us}|$. Recent analyses suggest that there is 3 σ discrepancy between the two results obtained from the method that uses the inclusive τ decay and the CKM unitarity constraint. While there might be a possibility that such a discrepancy could be explained by the effect of new physics, we should note that OPE yields a potential problematic uncertainty in the determination of $|V_{us}|$ from the inclusive hadronic τ decay by using the finite energy sum rule [a]. Thus, it is important to reduce the uncertainty of the QCD part, so that the so-called $|V_{us}|$ puzzle can be resolved.

In this report, for the purpose of resolving this problem, we would like to propose an alternative method of determining $|V_{us}|$, in which we use non-perturbative lattice QCD results for $\Pi(s)$ in addition to pQCD. Combining the two inputs, we expect that more reliable results could be obtained. In order to use lattice QCD inputs, we adopt a different weight function $\omega(s)$ that has poles in the Euclidean momentum region. As an illustrative example, we assume a weight function as $\omega(s) = \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$, where $-Q_k^2 < 0$ (for $k = 1, ..., N$), and $N \geq 3$. Taking $s_0 \to \infty$ in Eq.(1), we obtain

$$\int_0^{\infty} \rho(s)\omega(s)ds = \sum_k^N \mathrm{Res}\left(\Pi(-Q_k^2)\omega(-Q_k^2)\right). \qquad (3)$$

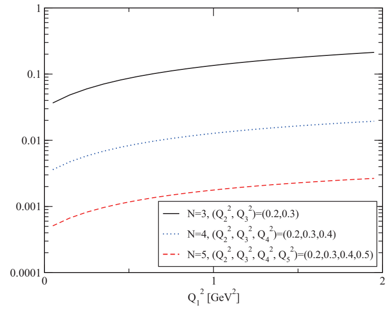

Fig. 1. Dependence of Q_1^2 on the ratio of the pQCD and kaon pole contribution. For pQCD result, $D = 0$ OPE ($N_f = 3$) and a conventional value of $|V_{us}|$ are used.

The lattice result is used for residues on the right hand side. The left hand side can be evaluated up to $s = m_\tau^2$ from experimental data, and we use a pQCD result for $s > m_\tau^2$. There is an advantage of using this method. As the above weight function $\omega(s)$ is highly suppressed in the high momentum region, the uncertainty introduced by pQCD can be reduced. In fact, Fig. 1 shows the dependence of weight function on the ratio of the OPE contribution of the spectrum integral in Eq.(3) to the contribution of the dominant kaon pole. As shown in Fig. 1, the OPE contribution can be suppressed by adding poles in the weight function.

As a preliminary study, we calculate $|V_{us}|$ determined from $\rho^{(0)}_A$. As for the lattice calculation of $\rho^{(0)}_A$, we use the lattice result of $L = 48$ near the physical quark mass[b]. Using a weight function with three poles of $(Q_1^2, Q_2^2, Q_3^2) = (0.1, 0.2, 0.3)$, we obtain a statistical relative error of 0.3%, which is competitive with previous results. As a future work, we will estimate systematic uncertainties such as lattice discretization, unphysical mass, and contributions from other channels, in particular pQCD effects.

References

1) P. A. Boyle et al., Int. J. Mod. Phys. Conf. Ser. **35**, 1460441 (2014) doi:10.1142/S2010194514604414 [arXiv:1312.1716 [hep-ph]].

† All the results shown here are preliminary.

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a) For a recent study of the inclusive τ decay by using the finite energy sum rule, see[1].

b) We thank RBC-UKQCD collaboration and Kim Maltman for providing lattice HVP and experimental data.