Chiral contamination in nucleon correlation functions†

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The study of excited-state contamination in nucleon correlation functions is motivated by lattice QCD computations. In particular, the Euclidean time dependence of two- and three-point functions is used to extract the nucleon mass and current matrix elements using standard lattice techniques. The long-time limit is used as a filter for the ground-state nucleon, such as exhibited in the effective mass of the two-point function, which has the behavior

$$M_{eff}(\tau) = \log \frac{G(\tau)}{G(\tau + a)} \to M_N + |Z'|^2 e^{-\Delta E\, \tau}, \quad (1)$$

where the exponential term arises from coupling to an excited state. A flat effective mass indicates saturation from the ground state, however, stochastically determined nucleon correlation functions exhibit a well-known signal-to-noise problem at long times. Without an increase in statistics, one can attempt to minimize $|Z'|^2$ through construction of better nucleon interpolating operators. Such operators have reduced coupling to excited states; and, to this end, we investigate the nature of pion-nucleon excited states using chiral perturbation theory. One should note that the signal-to-noise problem is considerably more detrimental to the case of three-point functions due to the necessity of two Euclidean time separations, and the lack of a positivity condition.

We have determined the pion-nucleon (also the pion-delta) contributions to nucleon two- and three-point correlators as a function of Euclidean time. Originally the computation was performed for a continuum of excited states[2)], and then revisited for a discrete set of states relevant for a torus[1)]. Strictly speaking, the nucleon operator of chiral perturbation theory is local, and the results are only applicable to extended lattice nucleon operators provided their characteristic size R is much smaller than the length scale introduced by the chiral symmetry breaking scale, $R \ll \Lambda_\chi^{-1}$. A discussion of this is contained in[3)], where the effective field theory mapping of lattice interpolating operators is also detailed.

To perform these computations of unamputated correlation functions using chiral perturbation theory, the crucial observation is that the results can be cast in the form

$$G_\chi(\tau) = |Z_O|^2 e^{-M_N \tau} \Big[1 + |Z_{\pi N}|^2 e^{-\Delta E_{\pi N} \tau} \Big], \quad (2)$$

where, for simplicity, we illustrate the case of the two-point function. The overlap $|Z_O|^2$ is determined by the short-distance dynamics of the nucleon interpolating operator. The relative overlap, $|Z_{\pi N}|^2$, however, is a long-distance correction that can be determined universally using chiral perturbation theory. The two-point function exhibits no surprises. Specializing to the case of the iso-vector, axial-vector current matrix element in the nucleon, we compute the ratio of three-point to two-point functions using chiral perturbation theory, namely

$$G_A(\tau, t) = \langle 0 | O_N(\tau) J^+_{\mu 5}(t) O^\dagger_N(0) | 0 \rangle \Big/ G_\chi(\tau), \quad (3)$$

and the result is shown in Figure 1. We find that the dominant excited-state contamination arises, not surprisingly, from nucleon to pion-nucleon intermediate states. For insufficient time separation, such excited-state contamination drives the axial-current correlation function upwards. On the basis of perturbative pion-nucleon interactions, we conclude that the consistent lattice underestimation of the nucleon axial charge, g_A, is not likely due to contamination from pion-nucleon states. Lattice QCD studies with well-localized interpolating operators would allow for confrontation with this prediction.

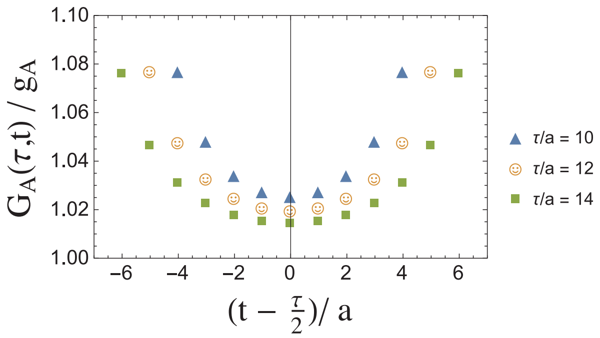

Fig. 1. Excited-state contamination in the axial-vector three-point function of the nucleon. The contamination is plotted as a function of the current-insertion time t (relative to the source-sink midpoint) for three values of the source-sink separation τ. The lattice spacing a is taken to be 0.1 fm. Excited states with pions tend to drive the axial correlator upwards, potentially leading to overestimation of the axial charge of a few percent.

References

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† Condensed from Reference[1)]

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