Perturbative matching for quasi-PDFs between continuum and lattice

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Parton distribution functions (PDFs) play an important role in understanding the structure of nucleons. In the search for the nucleon structure through experiment, the PDF is important; however it is currently estimated using the model assumption. Its direct lattice QCD calculation is impossible because it basically involves light-cone dynamics. In principle, the calculation can be made by operator product expansion; this method, however, is quite difficult because in the higher moments, the signal-to-noise ratios worsen. Recently, a strategy has been proposed, in which a computable quantity on the lattice (quasi-PDF) can be related to the (normal) PDF by perturbative matching.[1] The relation between the normal-PDF $q(x,\mu)$ and the quasi-PDF $\widetilde{q}(x,P_3,\Lambda)$ can be written as

$$\widetilde{q}(x,\Lambda,P_3)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_3},\frac{\Lambda}{P_3}\right)q(y,\mu)+\cdots, \quad (1)$$

where x is the momentum fraction of the parent hadron being carried by a quark, μ is the renormalization scale, and Λ is the UV cutoff scale. The quasi-PDF is presented by

$$\widetilde{q}(\tilde{x},\Lambda,P_3)=\int\frac{d\delta z}{2\pi}e^{-i\tilde{x}P_3\delta z}\langle\mathcal{N}(P_3)|O_{\delta z}|\mathcal{N}(P_3)\rangle, \quad (2)$$

where the matrix element (ME) is defined as the transition amplitude between the in- and out-nucleon states with momentum P_3 in the z-direction; the non-local operator elongated in the z-direction is described as

$$O_{\delta z}=\int_x\overline{\psi}(x+\hat{\mathbf{3}}\delta z)\gamma_3U_3(x+\hat{\mathbf{3}}\delta z;x)\psi(x), \quad (3)$$

with a Wilson line operator $U_3(x+\hat{\mathbf{3}}\delta z;x)$. The essential part in Eq. (2) is the nonperturbative ME, which we call quasi-PDF ME and write $\mathcal{M}_{\delta z}(P_3)$. This ME can be calculated using the lattice QCD simulation because it is not time-dependent. We need a matching to convert the ME obtained from the lattice calculation to the continuum counterpart, which has been omitted in the current lattice calculations and is a main purpose of this report.

To achieve matching between lattice and continuum, we have two choices: matching "in momentum space" or "in coordinate space". The first one is similar to the continuum matching in Eq. (1). In this approach, δz is first integrated out; thus, the z-component of the incoming and outgoing momentum at the vertex is fixed to be $\tilde{x}P_3$. However, we take the second approach, which is simple and more easily controllable in the actual lattice QCD simulation. The matching is achieved by a δz dependent matching factor:

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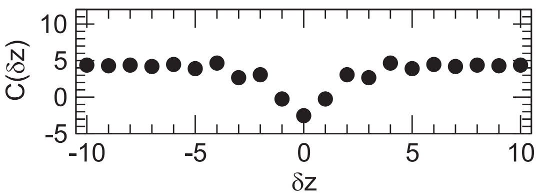

Fig. 1. One-loop coefficient of the matching factor.

$$[\mathcal{M}_{\delta z}(P_3)]^{\rm cont}=Z(\delta z)\,[\mathcal{M}_{\delta z}(P_3)]^{\rm latt}. \quad (4)$$

As the matching factor for operators does not depend on the external states, we can set $P_3=0$ for the calculation of the amplitude.

The transition amplitude involves power divergences, which arises from the Wilson line operator in both continuum and lattice. The power divergence must be subtracted nonperturbatively so that the theory is well-defined. There are several ways to perform the subtraction and we choose to use a static $Q\bar{Q}$ potential that shares the same power divergence. The Wilson line operator with a contour $\mathcal{C}$, $W_\mathcal{C}$, can be renormalized as

$$W_\mathcal{C}=e^{\delta m\ell(\mathcal{C})}W_\mathcal{C}^{\rm ren}, \quad (5)$$

where the superscript "ren" indicates that the operator is renormalized, $\ell(\mathcal{C})$ denotes the length along the contour $\mathcal{C}$, and δm denotes the mass renormalization of a test particle moving along the contour $\mathcal{C}$.[2] By choosing the renormalization condition for the static $Q\bar{Q}$ potential at a distance R_0 so that $V^{\rm ren}(R_0)=V_0$, the mass renormalization can be obtained as[3] $\delta m=(V_0-V(R_0))/2$. Using the renormalization condition above, we can define power divergence free ME as

$$\mathcal{M}^{\rm S}_{\delta z}(P_3)=e^{\frac{V(R_0)-V_0}{2}|\delta z|}\mathcal{M}_{\delta z}(P_3). \quad (6)$$

We only show the matching factor between continuum and lattice here using a naive fermion for simplicity, while it is not realistic for the actual simulation. We calculate it at one-loop order with HYP2 link smearing of the Wilson line and with mean-field improvement. Figure 1 shows the one-loop coefficient of the matching factor

$$Z(\delta z)=1+\frac{g^2}{(4\pi)^2}\frac{4}{3}C(\delta z)+O(g^4). \quad (7)$$

Using this, we can obtain the ME in continuum and the quasi-PDF can be calculated. We are currently planning to perform the actual simulation with this matching strategy.

References
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3) B. U. Musch et al., Phys. Rev. D **83**, 094507 (2011).