# Current Reflection and Transmission at Conformal Defects: Applying BCFT to Transport Process†

T. Kimura[*1,*2] and M. Murata[*3]

A powerful method for studying critical phenomena with conformal defects is boundary conformal field theory (BCFT). However, it has not been completely understood how BCFT describes the reflection/transmission at conformal defects. In this report, we define the reflection/transmission coefficient for conserved currents, as a natural generalization of that based on the energy-momentum tensor.[1)]

We consider two one-dimensional quantum systems connected by a junction, which can be considered as an impurity interacting with the bulk. Let us assume that the first system is in the positive domain $x > 0$, the second is in the negative $x < 0$, and they are connected at the origin as depicted in Fig. 1(a). Now we shall describe the above system in terms of BCFT. Corresponding to the two quantum systems, the BCFT picture involves two CFTs: $\mathrm{CFT}_1$ and $\mathrm{CFT}_2$. These CFTs are defined in the upper and lower half planes respectively as depicted in Fig. 1(b). The real axis, which divides the two CFTs, stands for the world line of the impurity, or the defect. We can reformulate this system to obtain $\mathrm{CFT}_1 \times \overline{\mathrm{CFT}}_2$ in the upper half plane thanks to the folding trick,[2)] as shown in Fig. 2. In this way, the junction of the one-dimensional quantum systems can be mapped into a CFT boundary condition.

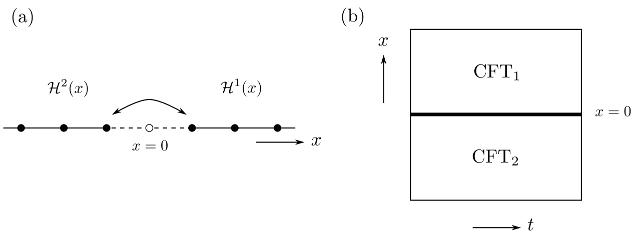


Fig. 1. From the *impurity* to the *defect*. (a) Two one-dimensional systems are connected through the impurity at $x = 0$. (b) Adding the time direction and taking the continuum limit, that system is mapped into the two-dimensional system with the defect along the line $x = 0$.

We assume that $\mathrm{CFT}_{1,2}$ have the same symmetry subalgebra $\mathcal{C}$, which is preserved at the conformal defect. For such a defect, we choose the following current gluing condition $\left(j_n^{\mathrm{tot},a} + \bar{j}_{-n}^{\mathrm{tot},a}\right)|B\rangle = 0$, where $j_n^{\mathrm{tot},a}$ takes values in the Kac–Moody algebra $\hat{\mathcal{C}}$, and $\bar{j}$ is the anti-holomorphic part. We then introduce the $R$-matrix based on this boundary state $|B\rangle$,

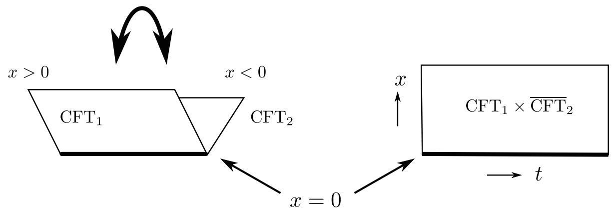


Fig. 2. From the *defect* to the *boundary*. By using the folding trick, a system with the defect is mapped into another system defined on the upper half plane with the boundary.

$$R[\mathcal{C}]^{ij,ab} = -\frac{\langle 0|j_1^{i,a}\bar{j}_1^{j,b}|B\rangle}{\langle 0|B\rangle} . \tag{1}$$

Since we have three constraints for this matrix, it has only one degree of freedom. Letting $\omega_B[\mathcal{C}]$ be

$$d^{ab}\omega_B[\mathcal{C}] = -\frac{1}{k_1 k_2 (k_1+k_2)}\frac{\langle 0|K_1^a \overline{K}_1^b|B\rangle}{\langle 0|B\rangle} , \tag{2}$$

the $R$-matrix is given by

$$R[\mathcal{C}] = \frac{k_1 k_2}{k_1+k_2}\left(\begin{pmatrix} \frac{k_1}{k_2} & 1 \\ 1 & \frac{k_2}{k_1} \end{pmatrix} + \omega_B[\mathcal{C}]\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right) , \tag{3}$$

where $d^{ab}$ is the Cartan–Killing form, $k_i$ is the level of the Kac–Moody algebra, and $K_n^a = k_2 j_n^{1,a} - k_1 j_n^{2,a}$. We can introduce the reflection and transmission coefficients based on this matrix:

$$\mathcal{R}[\mathcal{C}] = \frac{R^{11}+R^{22}}{k_1+k_2} , \quad \mathcal{T}[\mathcal{C}] = \frac{R^{12}+R^{12}}{k_1+k_2} , \tag{4}$$

which satisfy the current conservation $\mathcal{R} + \mathcal{T} = 1$. Applying this formula to the coset-type boundary state with $\mathcal{C} = su(2)$,[3)] we obtain

$$\mathcal{T}[su(2)] = \frac{2k_1 k_2}{(k_1+k_2)^2}\left(1 - \frac{S_{00}^{(k_1+k_2)} S_{\rho 1}^{(k_1+k_2)}}{S_{\rho 0}^{(k_1+k_2)} S_{01}^{(k_1+k_2)}}\right) , \tag{5}$$

with the modular $S$-matrix of $\mathrm{SU}(2)_k$ labeled by two integers, $S_{\rho\mu}^{(k)} = \sqrt{\frac{2}{k+2}}\sin\left(\frac{\pi}{k+2}(2\rho+1)(2\mu+1)\right)$.

References

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*1 RIKEN Nishina Center
*2 Institut de Physique Théorique, CEA Saclay
*3 Institute of Physics, ASCR