# Radion stabilization in the presence of Wilson line phase[†]

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In a higher-dimensional gravity theory, a scalar field $\phi$ called "radion" appears in the extra-dimensional graviton components, and its vacuum expectation value is related to the size of the extra space. The stabilization of the radius is crucial for the solution to the hierarchy problem in the Randall-Sundrum model[1)] and inflation based on the radion[2)]. The stabilization is realized by the quantum effects of a graviton and fermions in a 5D model $M_4 \times S^1$[3)].

In a higher-dimensional gauge theory, the extra-dimensional components of gauge bosons are massless at the tree level because of gauge invariance, and their zero modes become dynamical degrees of freedom called the Wilson line phases $\theta$ and are stabilized by quantum corrections[4)]. There is a possibility that realistic gauge symmetries including the standard model ones survive after the stabilization of the Wilson line phases. The Wilson line phase receives finite radiative corrections in its mass and can play the role of the Higgs boson[5)], providing a solution to the gauge hierarchy problem. An inflation model has been proposed based on the idea that the Wilson line phase becomes the inflaton[6)].

We investigate how the Wilson line phase and the Casimir energy from various bulk fields are involved in the stabilization of the radion in a different setup. Particularly, we study the stabilization of the extra-dimensional radius of $S^1$ in the presence of a Wilson line phase of the extra $U(1)$ gauge symmetry in 5D space-time with a flat background metric and without branes, by using the effective potential $V$ for $\phi$ and $\theta$ at the one-loop level.

Our model consists of the 5D graviton $\hat{g}_{MN}$, a $U(1)$ gauge boson $B_M$, $c_1$ charged fermions $\psi_i$ ($i = 1, \cdots, c_1$), and $c_2$ $U(1)$ neutral fermions $\eta_l$ ($l = 1, \cdots, c_2$). We take $M^4 \times S^1$ as the background 5D space-time and impose periodic boundary conditions on every field. We obtain the one-loop potential

$$V(\phi,\theta) = -\frac{6}{\pi^2}\frac{1}{\phi^2 L^4}\zeta(5) + c_2\frac{3}{\pi^2}\frac{1}{\phi^2 L^4}\left[\mathrm{Li}_5(e^{-L\mu\phi^{1/3}}) + L\mu\phi^{1/3}\mathrm{Li}_4(e^{-L\mu\phi^{1/3}}) + \frac{1}{3}L^2\mu^2\phi^{2/3}\mathrm{Li}_3(e^{-L\mu\phi^{1/3}})\right]$$
$$+c_1\frac{3}{\pi^2}\frac{1}{\phi^2 L^4}\mathrm{Re}\left[\mathrm{Li}_5(e^{-Lm\phi^{1/3}}e^{i\theta}) + Lm\phi^{1/3}\mathrm{Li}_4(e^{-Lm\phi^{1/3}}e^{i\theta}) + \frac{1}{3}L^2 m^2\phi^{2/3}\mathrm{Li}_3(e^{-Lm\phi^{1/3}}e^{i\theta})\right], \quad (1)$$

where $L$, $m$ and $\mu$ are the compactification circumference, the mass of the charged fermions and the neutral fermions, respectively.

The potential has a finite minimum in the presence of neutral fermions the number $c_2$ of which is larger than $c_1+2$ and the radion is stabilized at a certain finite value of $\phi$. A typical shape of $V(\phi,\theta)$ is depicted in Fig. 1. From this figure, we see that the true minimum of the potential is located on the line $\theta = \pi$. The values of $\phi$ and the potential at the minimum depend on the parameters $m$, $\mu$ and $L$. Their values do not drastically modify the shape of the potential drastically.

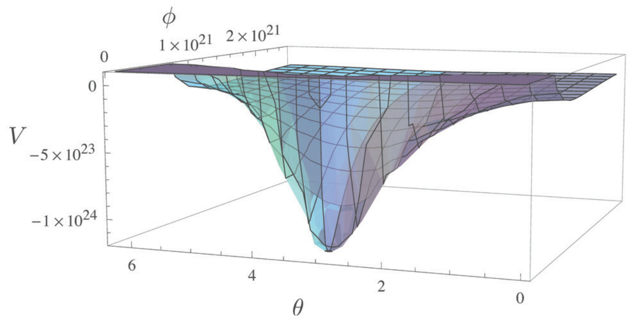


Fig. 1. The potential $V(\phi,\theta)$ with $c_1 = 1$, $c_2 = 4$, $m = 1\times10^{10}$GeV, $\mu = 1\times10^{10}$GeV and $L = 3\times10^{-17}\mathrm{GeV}^{-1}$.

In this work, we investigated the behavior of the potential for both large and small values of the radion and found that the potential does not have a finite minimum in the case with only charged fermions as matter fields. The radion stabilization is realized in the presence of neutral fermions whose number is larger than the number of charged ones by two.

The remaining subject is the application of our potential $V(\phi,\theta)$ to an inflation model. By identifing the extra-dimensional scalar component of the 5D gauge field and/or the scalar component of the 5D metric as the inflaton, we examine whether the potential reproduces realistic inflation parameters. The radion properties differ from thoset of the Wilson line phase.

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References
1) L. Randall and R. Sundrum: Phys .Rev. Lett. **83**, 3370 (1999).
2) A. Mazumdar: Phys. Lett. B**469**, 55 (1999).
3) Y. Fukazawa, T. Inami, and Y. Koyama: Prog. Theor. Exp. Phys. **2013** 021B01 (2013).
4) Y. Hosotani: Phys. Lett. B**126**, 309 (1983).
5) H. Hatanaka, T. Inami, and C.S. Lim: Mod. Phys. Lett. A**13**, 2601 (1998).
6) N. Arkani-Hamed, H.-C. Cheng, P. Creminelli, and L. Randall: Phys. Rev. Lett. **90**, 221302 (2003).