Entanglement entropy of de Sitter space α-vacua[†]

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The de Sitter space is a very interesting space-time. It is a solution of Einstein equation when cosmological constant dominates, and it is related to the inflationary stage of our universe, as well as to the current stage of accelerating universe. A peculiar property of the de Sitter space is that the de Sitter invariant vacuum is not unique; it has a one-parameter family of invariant vacuum states $|\alpha\rangle$, called α-vacua[1,2].

The α-vacua give very peculiar behavior for the two point functions in the de Sitter space; the two-point functions on α-vacua between points x and y contain not only the usual short distance singularity $\delta(|x-y|)$, where $|x-y|$ is the de Sitter invariant distance between x and y, but also contain very strange singularity such as $\delta(|x-\bar{y}|)$ and $\delta(|\bar{x}-y|)$, where $\bar{x}$ and $\bar{y}$ represent the antipodal points of x and y, respectively. Since antipodal points in the de Sitter space are not physically accessible due to the separation by a horizon, one cannot have an immediate reason to discard two-point functions containing such an antipodal singularity. It is therefore unclear which vacuum should be realized in our universe. As a result, a number of studies have been done on phenomenological aspects of the α-vacua (e.g. primordial perturbations generated during inflation).

Since which vacuum one should choose is always a very important question, one is motivated to calculate physical quantities not only in a particular vacuum but also in others, and see if there is a big reason to choose or discard a particular vacuum.

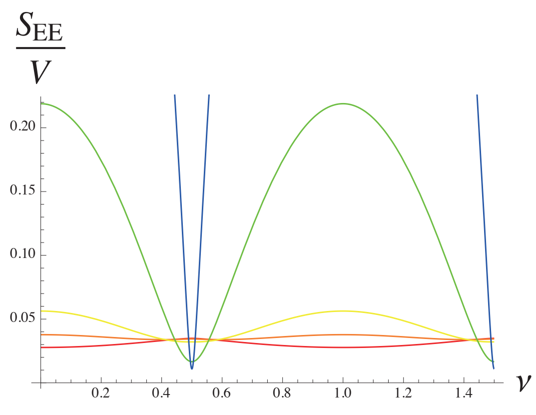

Fig. 1. Plot of S_{EE}/V against ν, for $\alpha = 0$ (red), 0.1 (orange), 0.25 (yellow), 1 (green) and 2 (blue). Notice the periodicity and reflection symmetries.

In this work, we computed the entanglement entropy in de Sitter α-vacua. By generalizing the recent calculation by Maldacena and Pimentel[3] in the Euclidean (or Bunch-Davies) vacuum for free scalar fields, we derived how entanglement entropy depends on α. The results are shown in Fig. 1 and Fig. 2.

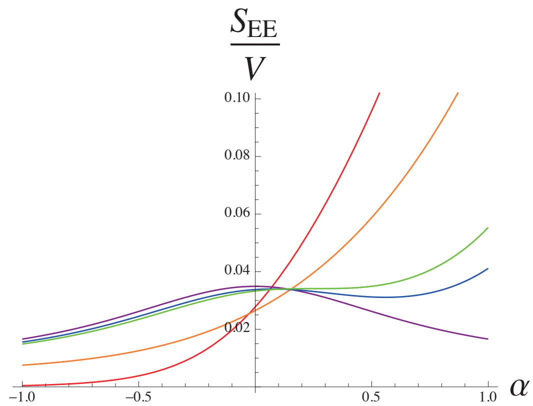

Fig. 2. Plot of S_{EE}/V against α, for $\nu = 0$ (red), 0.25 (orange), $\nu_c = 0.4062\ldots$ (green), 0.43 (blue) and 0.5 (purple).

As is seen in Fig. 2, the entanglement entropy increases significantly as we take α very large for generic values of ν. However only for $\nu = 1/2$ and $3/2$, this tendency disappears. Note that $\nu = 1/2$ is the conformal mass and $\nu = 3/2$ is massless. It is interesting to understand more physically why such a mass dependence occurs.

Our calculation is done for the free scalar field. Therefore direct comparison with the holographic calculation for the Euclidean vacuum[3] is difficult. It must be interesting to ask how the calculation of entanglement entropy on the α-vacua can be done in the strong coupling limit via holography, a la Ryu-Takayanagi formula[4]. Understanding these will hopefully shed more light on the question of which vacuum one should choose in the de Sitter space. We hope to come back to this question in near future.

References
1) E. Mottola, *Phys.Rev.* **D31** (1985) 754.
2) B. Allen, *Phys.Rev.* **D32** (1985) 3136.
3) J. Maldacena and G. L. Pimentel, *JHEP* **1302** (2013) 038, [arXiv:1210.7244].
4) S. Ryu and T. Takayanagi, *Phys.Rev.Lett.* **96** (2006) 181602, [hep-th/0603001].

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