# Update on lattice QCD calculation of neutral $B$ meson mixing in static limit of $b$ quark

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The Cabibbo–Kobayashi–Maskawa (CKM) matrix is a key part of elementary particle physics, and constraints on the elements $V_{ts}$ and $V_{td}$ can be obtained from $B^0 - \overline{B^0}$ mixing, where highly nonperturbative hadronic weak matrix elements play an essential role. We perform simulations on this subject using static heavy quark as the treatment of $b$ quark.[1)2)] The final values of $B$ meson decay constants and mixing matrix elements and $\xi$ parameter in the static $b$ quark limit are summarized as

$$f_B = 218.8(6.5)_{\rm stat}(16.1)_{\rm sys}\ [\rm MeV], \quad (1)$$
$$f_{B_s} = 263.5(4.8)_{\rm stat}(18.7)_{\rm sys}\ [\rm MeV], \quad (2)$$
$$f_{B_s}/f_B = 1.193(20)_{\rm stat}(35)_{\rm sys}, \quad (3)$$
$$f_B\sqrt{\hat{B}_B} = 240(15)_{\rm stat}(17)_{\rm sys}\ [\rm MeV], \quad (4)$$
$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 290(09)_{\rm stat}(20)_{\rm sys}\ [\rm MeV], \quad (5)$$
$$\xi = 1.208(41)_{\rm stat}(44)_{\rm sys}, \quad (6)$$

where statistical (stat) and systematic (sys) errors are given. In Fig. 1, the error budget is presented. Among the uncertainties presented in Fig. 1, a large portion of the error consists of statistical error, chiral extrapolation uncertainty, and renormalization uncertainty; it is thus important to reduce them to obtain more reliable results.

The first step in improving the current results is reducing the statistical errors using the All-Mode-Averaging (AMA) technique.[3)] This involves locating many source points in the measurement but using an approximation in obtaining quark propagators to greatly reduce computational cost. We show the preliminary results of the AMA calculation in Fig. 2 and compare them with the current results (without AMA). Although the AMA calculation is still ongoing, the presented error reduction is quite encouraging.

The systematic uncertainties involve chiral extrapolation and renormalization error. For the chiral extrapolation, we are currently focusing our effort on physical pion simulation, where we use RBC/UKQCD's 2+1 flavor $48^3 \times 96$ domain-wall fermion ensemble. By this calculation, most of the chiral extrapolation uncertainty is removed. Currently, the renormalization error is large for non-ratio quantities because only one-loop perturbation is employed. We are, however, investigating a possibility of nonperturbative method for the matching using RI/MOM scheme and coordinate space method,[4)] where removing power divergence is essential to the existence of the static heavy quark.

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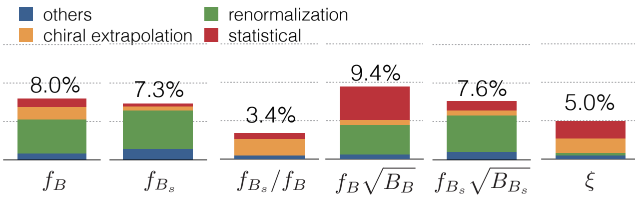


Fig. 1. Error budget for final quantities. The height of the bars indicates total error, whereas the relative size of the colors is determined by squared errors.[1)]

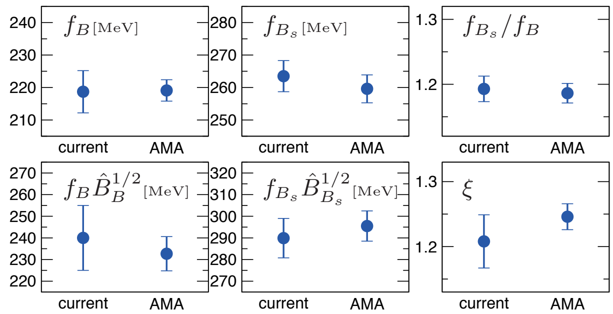


Fig. 2. Comparison of chiral and continuum extrapolated physical quantities between current and AMA results. The error denotes only statistical one.

Although the static limit is a good approximation for $b$ quark, it is known to have $O(\Lambda_{\rm QCD}/m_b) \sim 10\%$ uncertainty. To obtain precise results, we need to remove or reduce this uncertainty systematically. The heavy quark expansion of a heavy-light quantity $\Phi_{\rm hl}$, which has a finite asymptotic limit as $m_Q \to \infty$, is written as

$$\Phi_{\rm hl}(1/m_Q) = \Phi_{\rm hl}(0)\exp\left[\sum_{p=1}^{\infty}\gamma_p\left(\frac{\Lambda_{\rm QCD}}{m_Q}\right)^p\right], \quad (7)$$

where $m_Q$ is the heavy quark mass, which is heavier than the QCD scale $\Lambda_{\rm QCD}$. Our project finally includes simulations in light quark mass region (typically $c$ quark mass region) for obtaining expansion parameters $\gamma_p$s. Combining the static result with the lighter quark mass enables us to obtain results with much less uncertainty, where our static limit results play a crucial role as a valuable "anchor point".

References

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