Spin-orbit effects on pseudospin symmetry†

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Pseudospin symmetry (PSS)[1,2] was introduced in 1969 to explain the near degeneracy between pairs of nuclear single-particle states with the quantum numbers $(n-1, l+2, j = l+3/2)$ and $(n, l, j = l+1/2)$. They are regarded as the pseudospin doublets with modified quantum numbers $(\tilde{n} = n-1, \tilde{l} = l+1, j = \tilde{l} \pm 1/2)$. Although this concept was introduced for more than 40 years ago, the origin of PSS and its breaking mechanism in realistic nuclei have not been fully understood. Specifically, determining whether its nature is perturbative remains an unsolved problem.

Recently, we used the perturbation theory to investigate the symmetries of the Dirac Hamiltonian and their breaking in realistic nuclei[3], which provides a clear and quantitative way for investigating the perturbative nature of PSS. On the other hand, supersymmetric (SUSY) quantum mechanics can provide a PSS-breaking potential without singularity, and naturally interpret the unique feature that all states with $\tilde{l} > 0$ have their own pseudospin partners except for the intruder states[4]. Then, the similarity renormalization group (SRG) technique fills the gap between the perturbation calculations and the SUSY descriptions by transforming the Dirac Hamiltonian into a diagonal form and keeping every operator Hermitian[5,6].

Therefore, understanding the PSS and its breaking mechanism in a quantitative manner by combining the SRG technique, SUSY quantum mechanics, and perturbation theory is considered promising.

Here, we highlight the PSS-breaking potentials $\tilde{V}_{\rm PSO}(r)$, which are derived from the Dirac equation with the SRG and SUSY transformations.

In the upper panel of Fig. 1, the $\tilde{V}_{\rm PSO}(r)$ obtained without and with the spin-orbit (SO) term are shown for the $\tilde{f}$ orbitals. These potentials show several special features, which are crucial for understanding the PSS: (i) They are regular functions of r. (ii) Their amplitudes directly determine the sizes of reduced pseudospin-orbit (PSO) splittings $\Delta E_{\rm PSO} \equiv (E_{j_<} - E_{j_>})/(2\tilde{l}+1)$ according to the perturbation theory. (iii) Their shape, being negative at small radius but positive at large radius with a node at the surface region, can explain the general tendency of the PSO splittings becoming smaller with increasing single-particle energies.

To identify the SO effects, the $\tilde{V}_{\rm PSO}(r)$ obtained with the SO term is further decomposed into the contributions of the indirect and direct SO effects, because the former one represents the SO effects on $\tilde{V}_{\rm PSO}(r)$ via the superpotentials, while the latter is the SO potential itself. Comparison with the result obtained without the SO term shows that the indirect effect is small and eventually results in less influence due to the cancellation between the inner and outer regions. On the other hand, the SO potential is always positive with a peak at surface. It substantially raises the $\tilde{V}_{\rm PSO}(r)$, in particular for the surface region.

All of these properties are shown in the lower panel of Fig. 1, in which $\Delta E_{\rm PSO}$ are shown as a function of $E_{\rm av} = (E_{j_<} + E_{j_>})/2$. $\Delta E_{\rm PSO}$ match the amplitudes of $\tilde{V}_{\rm PSO}(r)$. The decreasing PSO splittings with increasing single-particle energies is due to the special shape of $\tilde{V}_{\rm PSO}(r)$. The SO term reduces $\Delta E_{\rm PSO}$ systematically, and this effect can be understood now in a quantitative manner.

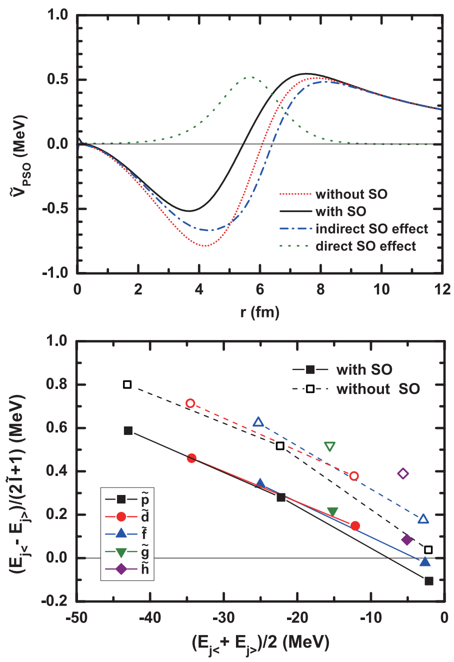

Fig. 1. Upper panel: PSS-breaking potentials $\tilde{V}_{\rm PSO}(r)$ obtained with and without SO term. The former one is decomposed into the indirect and direct the SO effects. Lower panel: $\Delta E_{\rm PSO}$ vs $E_{\rm av}$ with and without the SO term, where $j_<, j_>$ stand for the $\tilde{l} \mp 1/2$ states.

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