Non-Lagrangian theories from brane junctions †

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In a seminal article[1)], Gaiotto argued that a large class, called class $\mathcal{S}$, of $\mathcal{N}=2$ superconformal field theories (SCFT) in four dimensions (4D) can be obtained by a twisted compactification of a 6D $(2,0)$ SCFT on a Riemann surface of genus g with n punctures. The building blocks of the class $\mathcal{S}$ theories are tubes and pairs of pants that correspond to gauge groups and matter multiplets, respectively. Subsequently, a relation between the partition functions of the $\mathcal{N}=2$ $SU(N)$ gauge theories and the correlation functions of the 2D A_{N-1} Toda CFTs was proposed.[2)] Computation of 2-point and 3-point functions in a CFT would in principle yield a complete understanding of the n-point functions.

It is important to note that there is a fundamental difference between the $SU(2)$ and the $SU(N)$, $N>2$, cases. For the $SU(2)$ quiver gauge theories[2)] that are related to the 2D Liouville CFT, there is only one type of puncture on the Riemann surface and hence the Liouville CFT has only one class of 2D 3-point functions to be calculated. On the other hand, the $SU(N)$ case with $N>2$ has more than one kind of puncture. So far, the case with three special $SU(N)$ punctures T_N has remained elusive, since neither the T_N Nekrasov partition functions nor the Toda three-point correlators are known. The situation is further aggravated by the fact that the corresponding 4D theories do not have a Lagrangian description. Even though there is no known Lagrangian description of the 4D T_N theories, we are able to obtain the partition functions for their 5D uplift[3)] using topological strings on the dual geometry of the 5-brane junctions.

In this paper, we compute the Nekrasov partition functions of the T_N junctions as refined topological string partition functions.[4)] At this point, we make use of the quite recent conjecture of Iqbal and Vafa[5)] that says that the 5D superconformal index, which is the partition function on $S^4\times S^1$, can be obtained from the 5D Nekrasov partition function and thus from the topological string partition function

$$\mathcal{I}^{5D}=\int da\,|Z^{5D}_{\mathrm{Nek}}(a)|^2\propto\int da\,|Z_{\mathrm{top}}(a)|^2. \tag{1}$$

The E_6 superconformal index is obtained from the T_3 Nekrasov partition function by using the idea presented in Iqbal and Vafa,[5)] and we find that the results coincide with those of Kim et al.,[6)] computed via localization. When parallel external 5-brane legs appear in the toric web diagram, the corresponding partition functions contain extra degrees of freedom. In contrast to the massive spectrum in 5D, which forms a representation of the Wigner little group $SU(2)\times SU(2)$, referred to as the *full spin content representation*, these extra states do not transform as a correct representation under the Poincaré symmetry. Therefore, we call them *non-full spin content* contributions. We interpret this part as the contribution to the extra degrees of freedom appearing from the parallel 5-branes explained above. It should therefore be removed. To obtain the superconformal index from the topological string partition function, we have to eliminate all the non-full spin content from the partition function. Schematically, the partition function can be expressed as a sum of Young diagrams assigned to the product of strip geometries as

$$Z_{T_N}=\frac{1}{Z_{\text{non-full spin}}}\sum_{\boldsymbol{Y}}\prod_{i=1}^{N}Z_i^{\mathrm{strip}}(\boldsymbol{Y})\,. \tag{2}$$

The factor $Z_{\text{non-full spin}}$ is the BPS spectrum, which does not form a representation of the Poincaré symmetry, and Z^{strip} is the partition function of the strip geometry.

Finally, the 5D version of the AGTW relation, which suggests that the 5D Nekrasov partition functions are equal to the conformal block of q-deformed W_N Toda, implies the following relation between the superconformal index and the correlation functions of the corresponding q-deformed Toda field theory:

$$\mathcal{I}^{5D}(x,y)=\int[da]\left|Z^{\mathrm{5D}}_{\mathrm{Nek}}(a,m,\beta,\epsilon_{1,2})\right|^2$$
$$\propto\langle V_{\boldsymbol{\alpha}_1}(z_1)\cdots V_{\boldsymbol{\alpha}_n}(z_n)\rangle_{q\text{-Toda}}. \tag{3}$$

This is an important entry in the dictionary of the 5D/2D AGTW correspondence. The partition functions of the T_N brane junctions predict, up to an overall coefficient, the corresponding DOZZ formula for the three-point functions.

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