# Boundary Restoration of Chiral Symmetry†

B. Tiburzi*1

One of the hallmark features of the theory of strong interactions is spontaneous breaking of chiral symmetry. While quark masses explicitly break the chiral symmetry of the QCD action, the lightest quarks (up and down) have masses that can be treated as a perturbation about the symmetric $SU(2)_L \otimes SU(2)_R$ chiral limit. The formation of a chiral condensate by the QCD vacuum in the chiral limit, namely $<\overline{\psi}\psi> \neq 0$, spontaneously breaks the chiral symmetry down to the vector subgroup, $SU(2)_V$. This symmetry breaking pattern along with the explicit breaking due to the quark masses gives an explanation of the lightness of the iso-triplet of pseudo-scalar pions because they must be the emergent Goldstone bosons.

Lattice gauge theory provides a first principles method for solving QCD numerically on finite Euclidean space-time lattices. Strictly speaking, spontaneous symmetry breaking cannot occur in a finite volume. In practice, the formation of a chiral condensate on periodic lattices is determined by the size of the pion Compton wavelength compared to the lattice size[2–4]. In this work[1], we explore a different restoration of chiral symmetry. We consider the fate of chiral symmetry on a Euclidean manifold with three infinite directions, and one compact direction that, unlike the periodic case, has a boundary. Specifically the compact direction is subject to homogeneous Dirichlet boundary conditions (DBCs), as have been utilized recently in various lattice gauge theory computations.

The effect of a boundary on the chiral condensate cannot be ascertained within chiral perturbation theory, because the chiral condensate is determined by the expression

$$<\overline{\psi}\psi(x)> = -\frac{\Sigma}{4} < U(x) + U^{\dagger}(x) > + \cdots . \qquad (1)$$

Due to the unitarity of the coset manifold, the right-hand side of this relation does not vanish at the boundary in contradiction with the quark boundary conditions satisfied on the left-hand side. A consistent treatment of the chiral condensate in the presence of DBCs necessitates including the dynamics of isoscalar scalar mesons. For this reason, we employ the sigma model which shares the same symmetry breaking pattern as QCD with two light quark flavors, and provides the simplest model of spontaneous chiral symmetry breaking. The parameter $\Sigma$ in Eq. (1) becomes a field $\Sigma(x)$ satisfying DBCs. The chiral condensate $<\overline{\psi}\psi(x)>$ is determined by minimizing the action to find the vacuum configuration.

† Condensed from Reference[1]

*1 Department of Physics, The City College of New York (CUNY), USA; and RIKEN Nishina Center

An example of our results is shown in Fig. 1, which depicts the effect of DBCs on the volume-averaged value of the chiral condensate, defined by $\overline{<\overline{\psi}\psi>} = \frac{1}{L}\int_0^L dx < \overline{\psi}\psi(x)>$. In the limit of an asymptotically large extent $L$, the volume-averaged chiral condensate tends to the infinite volume value, however, the approach to asymptopia is slow. The asymptotic condensate can be determined in closed form, and averaged over the compact direction to produce

$$\overline{<\overline{\psi}\psi>} = <\overline{\psi}\psi> \left[1 - \frac{4\log 2}{m_\sigma L} + \cdots\right], \qquad (2)$$

from which we see power-law scaling controlled by the Compton wavelength of the sigma meson.

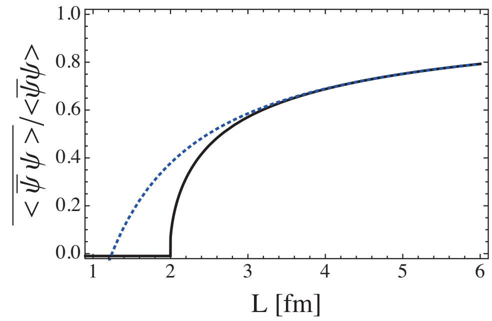


Fig. 1. Ratio of the volume-averaged condensate in the sigma model $\overline{<\overline{\psi}\psi>}$ to the infinite volume condensate $<\overline{\psi}\psi>$ plotted as a function of the finite extent $L$. The dotted curve shows the asymptotic formula in Eq. (2). Below $L \approx 2\,\mathtt{fm}$, the sigma model vacuum energy is minimized by $<\overline{\psi}\psi(x)> = 0$ for all $x$, and chiral symmetry is completely restored. The cusp is likely softened by including higher-lying sigma states.

Our sigma model results show that one should be cautious in interpreting results from lattice computations employing DBCs on small lattices. To establish the credibility of lattice computations with Dirichlet boundaries, one requires a lattice computation of the chiral condensate, either locally or volume averaged, which will ultimately reveal the extent to which chiral symmetry is restored in the presence of a boundary. In turn, frustration of the chiral condensate via Dirichlet boundaries may enable us to learn more about the mechanism that underlies spontaneous chiral symmetry breaking.

References

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