# Polarized fragmentation functions and electron-positron annihilation ${ }^{\dagger}$ 

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Electron-positron annihilation into (possibly polarized) hadrons, where one or more hadrons are identified, gives one access to fragmentation functions (FFs), which embody the process of a parton forming a hadron, and they contain important information about the strong interaction in the non-perturbative regime. (These are analogous to parton distribution functions (PDFs) that look at partons inside of hadrons.) Such experiments have been run by both the Belle Collaboration at KEK in Japan and the BABAR Collaboration at SLAC in the US. In order to have a complete framework to analyze FFs in electron-positron annihilation, one must write the cross section in a general form involving structure functions, which can then be calculated (for small transverse momentum of the exchanged boson) in terms of twist-2 transverse momentum dependent (TMD) FFs. For the future International Linear Collider (ILC), it is also beneficial to have complete results for polarized leptons including electroweak effects. Given its similarity to Drell-Yan, one should also be able to transcribe such results to that reaction, which would be useful for double-polarized Drell-Yan experiments at RHIC.

Here we extend earlier works ${ }^{1-5}$ ) in order to address these issues. The cross section for the reaction $e^{+} e^{-} \rightarrow$ $h_{a} h_{b} X$ is given by

$$
\begin{align*}
& 4 \frac{P_{a}^{0} P_{b}^{0} d \sigma}{d^{3} \vec{P}_{a} d^{3} \vec{P}_{b}}=\frac{2 \alpha_{e m}^{2}}{q^{2}}\left(L_{\mu \nu} W^{\mu \nu}\right)_{\gamma \gamma} \\
& \quad+\frac{M_{Z}^{4} G_{F}^{2}}{64 \pi^{2} q^{2}}\left(L_{\mu \nu} W^{\mu \nu}\right)_{Z Z} \\
& \quad \frac{\alpha_{e m} \sqrt{2} M_{Z}^{2} G_{F}}{8 \pi q^{2}}\left(\left(L_{\mu \nu} W^{\mu \nu}\right)_{\gamma Z}+h . c .\right) \tag{1}
\end{align*}
$$

where one has leptonic tensors $L^{\mu \nu}$ and hadronic tensors $W^{\mu \nu}$ for $\gamma$-exchange, $Z$-exchange, and their interference. As the present author, M. Schlegel, and A. Metz discuss in detail ${ }^{6)}$, one can write down the first term in (1) in terms of 72 structure functions using electromagnetic gauge invariance, hermiticity, and parity (see Eq. (3.21) of ${ }^{6}$ ). The second and third terms are more involved due to the fact that one no longer has the parity constraint. Nevertheless, one can calculate these terms (as well as the first) within the parton model at twist-2. The result leads to 128 structure functions (see Eqs. (4.34), (4.35) and Appendix A of ${ }^{6)}$ ). Where relevant, we have checked our results with those in ${ }^{1-4)}$. Thus, we have for the first time a complete framework for the study of TMD FFs within $e^{+} e^{-} \rightarrow h_{a} h_{b} X$ including electroweak terms and the

[^0]polarization of all particles.
We also note that if we make the replacements $\left(4 P_{a}^{0} P_{b}^{0} d \sigma / d^{3} \vec{P}_{a} d^{3} \vec{P}_{b}\right)_{e^{+} e^{-}} \longrightarrow\left(4 l^{0} l^{\prime 0} d \sigma / d^{3} \vec{l} d^{3} \vec{l}^{\prime}\right)_{D Y}$, $z_{a}\left(z_{b}\right) \rightarrow x_{a}\left(x_{b}\right)$, and $N_{c} \rightarrow 1 / N_{c}$, the structure functions associated with unpolarized leptons for the pure electromagnetic case are the same as those given in Drell-Yan ${ }^{5)}$ with the TMD PDFs replaced by their TMD FF analogues. The only additional change one must remember is that $h_{a}\left(h_{b}\right)$ in the $e^{+} e^{-}$case has a large minus- (plus-) component of momentum, whereas for Drell-Yan one normally uses the reverse convention. Along the same lines, one can easily transcribe the results in Appendix A of ${ }^{6)}$ to obtain the relevant expressions for Drell-Yan when one allows the $q \bar{q}$ pair to annihilate into a $Z$-boson. Thus, we have for the first time full results for double-polarized Drell-Yan that include electroweak effects, which would be needed if such experiments were conducted at RHIC.

We would finally like to highlight a structure function that appears in ${ }^{6)}$ for first time:

$$
\begin{equation*}
G_{U U}^{\mathrm{cos} 2 \phi, e w}=\sum_{q} \mathcal{F}_{5}^{q-}(s) C_{e w}^{q}\left[w_{3} H_{1}^{\perp} \bar{H}_{1}^{\perp}\right] \tag{2}
\end{equation*}
$$

where $\mathcal{F}_{5}^{q-}(s)$ is a prefactor that is nonzero if $Z$ exchange is included, $C_{e w}^{q}[\cdots]$ is the convolution of a weight $w_{3}$ and the Collins function $H_{1}^{\perp}$. One could access (2) at a future ILC and extract the Collins function. This result could then be checked against the Collins function that has been obtained recently from Belle and BABAR data ${ }^{7)}$. Given that the ILC would have around two orders of magnitude higher center-ofmass energy than Belle and BABAR, such an analysis would also be an important test of the TMD evolution formalism and its application to phenomenology, which has been of recent interest.

## References

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